

**What Is Claimed Is:**

- 1           1.       A method for automatically computing a derivative of a numerical  
2 expression within a digital computer system, comprising:  
3           receiving a representation of the numerical expression within the digital  
4 computer system, wherein the numerical expression includes one or more  
5 independent variables;  
6           forming an expression tree for the derivative of the numerical expression  
7 with respect to an independent variable, wherein the expression tree makes use of  
8 temporary variables to form results of sub-expressions for computing the  
9 derivative of the numerical expression; and  
10          wherein forming the expression tree involves seeking to introduce only  
11 temporary variables and associated sub-expressions as necessary to eliminate  
12 repeated common sub-expressions, thereby substantially minimizing the number  
13 of temporary variables; and  
14          using the expression tree to compute the derivative of the numerical  
15 expression during a computation.
- 1           2.       The method of claim 1, wherein for each temporary variable in the  
2 expression tree, the method forms an expression for a partial derivative of the  
3 temporary variable with respect to each independent variable.
- 1           3.       The method of claim 2, wherein forming the expression for a given  
2 partial derivative involves defining new temporary variables and corresponding  
3 sub-expressions and new partial derivatives.

1           4.     The method of claim 1, wherein a given sub-expression can  
2 include a multinomial that contains more than one binary operation.

1           5.     The method of claim 1, further comprising pruning sub-  
2 expressions and associated temporary variables that are not used by the  
3 computation.

1           6.     The method of claim 5, wherein pruning sub-expressions involves:  
2 marking all temporary variables used in evaluating the expression tree; and  
3 deleting unmarked temporary variables and associated sub-expressions.

1           7.     The method of claim 1,  
2 wherein the method is performed within a compiler; and  
3 wherein the representation of the numerical expression is in the form of an  
4 Abstract Syntax Tree (AST).

1           8.     The method of claim 1,  
2 wherein the method is performed within a pre-compiler;  
3 wherein the expression tree is in the form of a source code list; and  
4 wherein each temporary variable and associated sub-expression in the  
5 expression tree is represented by computer code that sets the temporary variable  
6 equal to the associated sub-expression.

1           9.     The method of claim 1, wherein the method is performed by code  
2 within a code library.

1           10.    The method of claim 1,

2 wherein the computation involves interval arithmetic; and  
3 wherein the one or more independent variables are interval variables.

1 11. A computer-readable storage medium storing instructions that  
2 when executed by a computer cause the computer to perform a method for  
3 automatically computing a derivative of a numerical expression within a digital  
4 computer system, the method comprising:  
5 receiving a representation of the numerical expression within the digital  
6 computer system, wherein the numerical expression includes one or more  
7 independent variables;  
8 forming an expression tree for the derivative of the numerical expression  
9 with respect to an independent variable, wherein the expression tree makes use of  
10 temporary variables to form results of sub-expressions for computing the  
11 derivative of the numerical expression; and  
12 wherein forming the expression tree involves seeking to introduce only  
13 temporary variables and associated sub-expressions as necessary to eliminate  
14 repeated common sub-expressions, thereby substantially minimizing the number  
15 of temporary variables; and  
16 using the expression tree to compute the derivative of the numerical  
17 expression during a computation.

1 12. The computer-readable storage medium of claim 11, wherein for  
2 each temporary variable in the expression tree, the method forms an expression  
3 for a partial derivative of the temporary variable with respect to each independent  
4 variable.

1           13.    The computer-readable storage medium of claim 12, wherein  
2   forming the expression for a given partial derivative involves defining new  
3   temporary variables and corresponding sub-expressions and new partial  
4   derivatives.

1           14.    The computer-readable storage medium of claim 11, wherein a  
2   given sub-expression can include a multinomial that contains more than one  
3   binary operation.

1           15.    The computer-readable storage medium of claim 11, wherein the  
2   method further comprises pruning sub-expressions and associated temporary  
3   variables that are not used by the computation.

1           16.    The computer-readable storage medium of claim 15, wherein  
2   pruning sub-expressions involves:  
3       marking all temporary variables used in evaluating the expression tree; and  
4       deleting unmarked temporary variables and associated sub-expressions.

1           17.    The computer-readable storage medium of claim 11,  
2       wherein the method is performed within a compiler; and  
3       wherein the representation of the numerical expression is in the form of an  
4   Abstract Syntax Tree (AST).

1           18.    The computer-readable storage medium of claim 11,  
2       wherein the method is performed within a pre-compiler;  
3       wherein the expression tree is in the form of a source code list; and

1            wherein each temporary variable and associated sub-expression in the  
2 expression tree is represented by computer code that sets the temporary variable  
3 equal to the associated sub-expression.

1            19.    The computer-readable storage medium of claim 11, wherein the  
2 method is performed by code within a code library.

1            20.    The computer-readable storage medium of claim 11,  
2 wherein the computation involves interval arithmetic; and  
3 wherein the one or more independent variables are interval variables.

1            21.    An apparatus for automatically computing a derivative of a  
2 numerical expression within a digital computer system, comprising:  
3            a receiving mechanism that is configured to receive a representation of the  
4 numerical expression within the digital computer system, wherein the numerical  
5 expression includes one or more independent variables;  
6            an expression tree forming mechanism that is configured to form an  
7 expression tree for the derivative of the numerical expression with respect to an  
8 independent variable, wherein the expression tree makes use of temporary  
9 variables to form results of sub-expressions for computing the derivative of the  
10 numerical expression; and  
11           wherein the expression tree forming mechanism seeks to introduce only  
12 temporary variables and associated sub-expressions as necessary to eliminate  
13 repeated common sub-expressions, thereby substantially minimizing the number  
14 of temporary variables; and  
15           an execution mechanism that is configured to use the expression tree to  
16 compute the derivative of the numerical expression during a computation.

1           22.    The apparatus of claim 21, wherein for each temporary variable in  
2 the expression tree, the expression tree forming mechanism is configured to form  
3 an expression for a partial derivative of the temporary variable with respect to  
4 each independent variable.

1           23.    The apparatus of claim 22, wherein in forming the expression for a  
2 given partial derivative, the expression tree forming mechanism is configured to  
3 define new temporary variables and corresponding sub-expressions and new  
4 partial derivatives.

1           24.    The apparatus of claim 21, wherein a given sub-expression can  
2 include a multinomial that contains more than one binary operation.

1           25.    The apparatus of claim 21, further comprising a pruning  
2 mechanism that is configured to prune sub-expressions and associated temporary  
3 variables that are not used by the computation.

1           26.    The apparatus of claim 25, wherein the pruning mechanism is  
2 configured to:  
3       mark all temporary variables used in evaluating the expression tree; and to  
4       delete unmarked temporary variables and associated sub-expressions.

1           27.    The apparatus of claim 21,  
2       wherein the apparatus resides within a compiler; and  
3       wherein the representation of the numerical expression is in the form of an  
4       Abstract Syntax Tree (AST).

1           28.    The apparatus of claim 21,  
2           wherein the apparatus resides within a pre-compiler;  
3           wherein the expression tree is in the form of a source code list; and  
4           wherein each temporary variable and associated sub-expression in the  
5 expression tree is represented by computer code that sets the temporary variable  
6 equal to the associated sub-expression.

1           29.    The apparatus of claim 21, wherein the apparatus includes code  
2 within a code library.

1           30.    The apparatus of claim 21,  
2           wherein the computation involves interval arithmetic; and  
3           wherein the one or more independent variables are interval variables.